Dynamics of Neutrino Oscillations and the Cosmological Constant Problem

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Abstract

The cosmological constant problem continues to represent a major challenge for theoretical physics and cosmology. The main difficulty arises from the large numerical discrepancy between observational limits of the cosmological constant and quantum predictions based on gravitational effects of the vacuum energy. In this work we argue that the experimental value of this constant may be recovered from the dynamics of neutrino oscillations.

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1. Introduction

The cosmological constant problem (c.c.p) represents one of the major unresolved issues of contemporary physics [1]. A numerical discrepancy of about 120 orders of magnitude exists between the theoretical and observational values of this constant [2]. It is presumed that a presently unknown symmetry operates in such a way as to enforce a vanishingly small constant while remaining consistent with all accepted field-theoretic principles. Several competing models attempting to resolve the c.c.p coexist (for a review of some of the relevant literature on the subject see [3-5] and references listed in [5]).
In this paper we argue that the observational value of the cosmological constant emerges from the physics of neutrino oscillations. Although the idea is not new (see e.g. [6] and references therein), our treatment is built on a much simpler foundation. Unlike the approach taken by previous authors, the model discussed here is exclusively built on the relativistic dynamics of massive fermions, as expressed by the standard Pontercovo formula [7], [15]. The paper is organized as follows: section 2 highlights the quantum field viewpoint on the cosmological vacuum and its difficulties. The derivation of the cosmological constant from the dynamics of neutrino oscillations is discussed in section 3. Concluding remarks are outlined in the last section.

2. Neutrino oscillations as vacuum fluctuations on a cosmic scale

The quantum origin of the c. c. p arises because the zero-point vacuum energy diverges quadratically in the presence of gravitation. Standard Quantum Field Theory in Minkowski space-time discards the zero-point vacuum energy through the use of a normal time-ordering procedure [8]. Because vacuum energy gravitates and couples to all other field energies present at the quantum level, cancellation of the zero-point term is no longer possible when gravitation produces measurable effects.

We believe that the large discrepancy between theory and observations in c.c.p is rooted in the fundamental incompatibility of Quantum Field Theory and General Relativity with regard to the very interpretation of the concept of vacuum. The vacuum of General Relativity (v-GR) represents a state devoid of matter and energy on the macroscopic scale, whereas the vacuum of Quantum Field Theory (v-QFT) is associated with the zero-point energy of all fields present at the quantum level. Thereby, in order to avoid the pitfalls of a full-blown quantum interpretation of the cosmological constant problem [14], we start from a different perspective and posit that neutrino oscillations represent experimental evidence for fluctuations of v-GR. This argument is supported by the fact that, according to our current knowledge, neutrinos are the lightest and the most stable lepton states and that they are ultra-weakly coupled to ordinary matter.

3. Neutrino oscillations and the cosmological constant

We start from the system of coupled first-order equations describing the evolution of neutrino mass eigenstates in matter [7]

$$\begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta(t)}{4iE} & -\dot{\theta}_m(t) \\ \dot{\theta}_m(t) & \dfrac{\Delta(t)}{4iE} \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} \tag{1}$$

Here, $\nu_{1,2}^m(t)$ denote the neutrino mass eigenstates of energy E, $\Delta^2(t) \equiv \mu_2^2(t) - \mu_1^2(t)$ represents the time-dependent mass-squared difference in matter and $\theta_m(t)$ is the effective mixing angle in matter. The following relations hold

$$\Delta(t)^2 \equiv \mu_2^2(t) - \mu_1^2(t)$$

$$\mu_{1,2}^2(t) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_\alpha) \mp \frac{1}{2}\sqrt{(\Delta m_{12}^2 \cos 2\theta - A)^2 + (\Delta m_{12}^2 \sin 2\theta)^2}$$

$$\tan 2\theta_m(t) = \frac{\Delta m_{12}^2 \sin 2\theta}{\Delta m_{12}^2 \cos 2\theta - A(t)} \tag{2}$$

$$\dot{\theta}_m = \frac{\Delta m_{12}^2 \sin 2\theta}{2\Delta(t)^2} \dot{A}$$

where $\mu_{1,2}(t)$ stands for the effective neutrino mass in matter, $\Delta m_{12}^2 = m_1^2 - m_2^2$ is the mass-squared difference in vacuum, $V_e(t)$ and $V_\alpha(t)$ are the effective interaction potentials for electron neutrino ν_e and the flavor eigenstate ν_α, respectively. $A(t)$ is given by

$$A(t) \equiv 2E[V_e(t) - V_\alpha(t)] \tag{3}$$

Two opposite cases are of interest for analysis. When

$$\frac{\Delta(t)}{4E} > \dot{\theta}_m(t) = \frac{\Delta m_{12}^2 \sin 2\theta}{2\Delta(t)^2}\dot{A} \tag{4}$$

the mass eigenstates $\nu_{1,2}^m(t)$ mix in small amounts to produce the flavor eigenstates and this transition is referred to as adiabatic. Many neutrino oscillations take place in the adiabatic regime with few mixing events. Conversely, when (4) is not satisfied, the transition is said to be non-adiabatic. To simplify the ensuing derivation and without a significant loss of generality, we limit the analysis to propagation in low-density matter and expand both the mass-splitting $\Delta(t)$ and time-rate of the effective mixing angle $\dot{\theta}_m(t)$ as follows

$$\begin{aligned} \Delta(t) &= \Delta m_{12} + P(t) \\ \dot{\theta}_m(t) &= \varepsilon + Q(t) \end{aligned} \tag{5}$$

in which $\varepsilon \ll 1$. The planar system of evolution equations (1) decouples into a time-independent and a time-dependent part

$$\begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta m_{12}}{4iE} & -\varepsilon \\ \varepsilon & \dfrac{\Delta m_{12}}{4iE} \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} + \begin{pmatrix} -\dfrac{P(t)}{4iE} & -Q(t) \\ Q(t) & \dfrac{P(t)}{4iE} \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} \tag{6}$$

Local stability analysis of (6) indicates that the trivial equilibrium $\nu_1^m = \nu_2^m = 0$ represents an elliptic point since

$$Tr\begin{pmatrix} -\dfrac{\Delta m_{12}}{4iE} & -\varepsilon \\ \varepsilon & \dfrac{\Delta m_{12}}{4iE} \end{pmatrix} = 0 \tag{7}$$

The stability analysis is representative for the dynamics of the conservative system (6) near equilibrium because [9]

$$\det\begin{pmatrix} -\frac{\Delta m_{12}}{4iE} & -\varepsilon \\ \varepsilon & \frac{\Delta m_{12}}{4iE} \end{pmatrix} = \frac{\Delta m_{12}^2}{16E^2} + \varepsilon^2 \neq 0 \tag{8}$$

It is thus reasonable to assume that the time-dependent contribution corresponding to neutrino propagation in low-density matter may be neglected up to a first-order approximation. The time-independent term of (6) may be shown to be formally identical to the equation of a linear harmonic oscillator [10], that is

$$\ddot{\nu}_1^m + [\frac{\Delta m_{12}^2}{16E^2} + \varepsilon^2]\nu_1^m = 0 \tag{9}$$

Stated differently, if E sets the natural energy scale of neutrino physics in low-density matter, flavor transitions governed by (6) may be interpreted as harmonic oscillations of the mass eigenstate $\nu_1^m(t)$ with proper frequency and Compton wavelength

$$\begin{gathered} \omega_{12} = \Delta m_{12}/4 \\ \lambda_C = (\Delta m_{12}/4)^{-1} \end{gathered} \tag{10}$$

Assuming that there is roughly one quasi-particle of mass Δm_{12} per the unit volume defined by the Compton wavelength cubed (λ_c^3), we derive the following expectation value for the cosmological vacuum density

$$\rho_v \sim \frac{\Delta m_{12}}{4\lambda_c^3} = \frac{1}{256}\Delta m_{12}^4 \tag{11}$$

Current testing data indicate that the solar neutrino mass splitting falls in the range [11]

$$\Delta m_{12,sol}^2 < 9.5\times10^{-5} eV^2 \tag{12}$$

Using (11) and (12) leads to the "first-order" estimate

$$\boxed{\rho_v^{sol} < 3.525\times10^{-11} eV^4} \tag{13}$$

This value agrees well with the recent observational bound of the cosmological constant according to which [5, 12]

$$\rho_\Lambda = \Lambda/8\pi G \leq 1.6\times10^{-11} eV^4 \tag{14}$$

4. Concluding remarks

The starting point of this brief report was setting a primary distinction between the vacuum of General Relativity (v-GR) and the zero-point energy concept of QFT (v-QFT). Neutrino oscillations were interpreted as direct evidence for fluctuations of (v-GR). Following this route, we were led to a numerical prediction that agrees well with current observational data. Analysis of neutrino oscillations in low-density matter has been carried out in the framework of Pontercovo's model and without any reference to vacuum energy density of quantum fluctuations.

5. References

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